77. We may regard an aspect A, B, C, ... in the aggregate as a single unit V, which may be termed a *unified aspect* of a, b, c, ...

Section 167 is erroneous, and should be omitted.

I may add the following errata:—

In Sec. 69, line 3,	for "undistinguished"	read "distinguished."
,, ,, 122, ,, 1,	,, "aspects"	,, "collections."
,, page 43, footnote,	,, "Grassman"	,, "Grassmann."
,, ,, 56 ,,	,, "Pierce"	,, "Peirce."
,, Sec. 385, line 8,	,, "m units"	,, "r units."

III. "On Ellipsoidal Current Sheets." By Horace Lamb, M.A., F.R.S., Professor of Pure Mathematics in the Owens College, Victoria University, Manchester. Received March 2, 1887.

(Abstract.)

This paper treats of the induction of electric currents in an ellipsoidal sheet of conducting matter whose conductivity per unit area varies as the perpendicular from the centre on the tangent plane, or (say) in a thin shell of uniform material bounded by similar and coaxial ellipsoids. The method followed is to determine in the first instance the normal types of free currents. In any normal type the currents decay according to the law $e^{-t/\tau}$; the time-constant τ may be conveniently called the "modulus of decay," or the "persistency" of the type.

When the normal types and their persistencies have been found, it is an easy matter to find the currents induced by given varying electromotive forces, assuming these to be resolved by Fourier's theorem, as regards the time, into a series of simple harmonic terms. Supposing then that we have an external magnetic system whose potential varies as e^{ipt}, we can determine a fictitious distribution of current over the shell, which shall produce the same field in the interior. If $\bar{\phi}$ denote the current-function for that part of this distribution which is of any specified normal type, ϕ that of the induced currents of this type, it is shown that

$$\phi = -\frac{ip\tau}{1+ip\tau}\bar{\phi},$$

where τ is the corresponding persistency of free currents. When $p\tau$ is very great this becomes

$$\phi = -\bar{\phi},$$

in accordance with a well-known principle.

This method can be applied to find the currents induced by rota-

The Royal Society is collaborating with JSTOR to digitize, preserve, and extend access to *Proceedings of the Royal Society of London.* www.jstor.org

tion of the shell in a constant field, it being known from Maxwell's 'Electricity,' § 600, that the induced currents are the same if we suppose the conductor to be fixed, and the field to rotate in the opposite direction. When the conductor is symmetrical about the axis of rotation, the current-function of any normal type contains as a factor cos $s\omega$ or sin $s\omega$, where ω is the azimuth, and s is integral (or zero). When we apply Maxwell's artifice, the corresponding time-factor is e^{ispt}, where p is the angular velocity of the rotation; and we easily find that the system of nduced currents of any normal type is fixed in space, but is displaced relatively to the field though an angle

$$\frac{1}{s} \text{ arc tan } ps\tau$$

in azimuth, in the direction of the rotation.

In the most important normal types the distribution of current over the ellipsoid is one which has been indicated by Maxwell ('Electricity,' § 675) as giving a uniform magnetic field throughout the interior. For instance, the axes of coordinates being along the principal axes, a, b, c, we may have

$$\phi = \mathrm{C}z, \quad \ldots \ldots \ldots \quad (1.)$$

and the corresponding persistency is

$$\tau = (4\pi - \mathrm{N})\frac{\epsilon}{\rho}\frac{a^2b^2}{a^2+b^2}, \quad \ldots \ldots \quad (2.)$$

where $$\mathrm{N} = 2\pi abc\int_0^\infty \frac{d\lambda}{(a^2+\lambda)^{\frac{1}{2}}(b^2+\lambda)^{\frac{1}{2}}(c^2+\lambda)^{\frac{3}{2}}}, \quad \ldots \quad (3.)$$

ρ denoting the specific resistance of the material, and ϵ the small constant ratio of the thickness of the shell to the perpendicular on the tangent plane. There is a difference of electric potential over the shell, viz., we have

$$\psi = \mathrm{A}xy, \quad \ldots \ldots \ldots \ldots \quad (4.)$$

where $$\mathrm{A} = \frac{\mathrm{L}a^2 - \mathrm{M}b^2}{(a^2+b^2)\tau}\mathrm{C},$$

L, M being obtained from (3) by interchanging a and c, or b and c, respectively. This implies a certain distribution of electricity over the outer surface of the shell.

Some special forms of the ellipsoid (*e.g.*, a sphere, or an elliptic cylinder) are considered, and the formula (2) shown to agree with the results obtainable, in these cases, in other ways.

The problem of induced currents due to simple harmonic variation

of a uniform field, or to rotation of the shell in a uniform and constant field, is then solved; and the results are found to agree with the general theory above sketched.

In the higher normal types the current-function ϕ is a Lamé's function, degenerating into a spherical harmonic when two of the axes of the ellipsoidal shell are equal. This case alone is further discussed in the present paper; the persistency of each normal type is found, and various particular cases are considered. Of the special forms which the conductor may assume, the most interesting is that in which the third axis (that of symmetry) is infinitesimal, so that we have practically a circular *disk*, whose resistance ρ' per unit area varies according to the law

$$\rho' = \rho_0' \sqrt{\{1-r^2/a^2\}}, \quad \ldots \ldots \quad (5.)$$

where ρ_0' is the resistance at the centre, a is the radius, and r denotes the distance of any point from the centre. In any normal type of free currents the current-function is of the form

$$\phi = \mathrm{C} \cdot (1-\mu^2)^{\frac{1}{2}s} \frac{d^s \mathrm{P}_n(\mu)}{d\mu^s} \begin{Bmatrix} \cos \\ \sin \end{Bmatrix} s\omega, \quad \ldots \ldots \quad (6.)$$

where $\qquad \mu = \sqrt{\{1-r^2/a^2\}},$

provided $n-s$ be *odd*; in other words, the current lines are the orthogonal projections on the plane of the disk of the contour-lines of a zonal ($s = 0$) or tessaral harmonic, drawn on the surface of a concentric sphere of radius a. The corresponding persistency is

$$\tau = \frac{\pi^2 a}{\{n(n+1)-s^2\}\rho_0'} \cdot \frac{\underline{|n-s}}{\underline{|n+s}} \cdot \left\{ \frac{1 \cdot 3 \ldots (n+s)}{2 \cdot 4 \ldots (n-s-1)} \right\}^2. \quad (7.)$$

In the most persistent type of free currents we have $n = 1$, $s = 0$, and therefore

$$\tau = \frac{\pi^2 a}{2\rho_0'}.$$

This result is of some interest, as showing that the electrical time-constant for a disk of *uniform* resistance ρ_0' must at all events be considerably less than $4{\cdot}93\ a/\rho_0'$.*

* I find by methods similar to those employed by Lord Rayleigh for the approximate determination of various acoustical constants, that the true value lies between $\pi a/\rho'$ and $2{\cdot}26\ a/\rho'$. For a disk of copper ($\rho = 1600$ C.G.S.), whose radius is a decimetre and thickness a millimetre, the lower limit gives 0·0014 sec. For disks of other dimensions the result will vary as the radius and the thickness conjointly. I hope shortly to publish the details of the investigation on which these estimates are founded.

The problem of induced currents is then discussed, and I consider more particularly the case of a circular disk, of the kind indicated, rotating in any constant magnetic field. In view of the physical interest attaching to the question, it would be interesting to have a solution for the case of a *uniform* disk; but in the absence of this, the solution for the more special kind of disk here considered may not be uninstructive.

As in all our calculations relating to ellipsoids of revolution, we employ elliptic coordinates; viz., seeking the origin at the centre of the disk, and the axis of z perpendicular to its plane, we write

$$\left.\begin{aligned} x &= a\sqrt{(1-\mu^2)}\sqrt{(\zeta^2+1)}\cos\omega \\ y &= a\sqrt{(1-\mu^2)}\sqrt{(\zeta^2+1)}\sin\omega \\ z &= a\mu\zeta. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (8.)$$

where μ may range from 1 to 0, and ζ from zero (its value at the disk) to ∞. The magnetic potential $\overline{\Omega}$ due to the field may be supposed expanded, for the space near the disk, in a series of terms of the form

$$\overline{\Omega} = \overline{\mathrm{A}}\,(1-\mu^2)^{\frac{1}{2}s}\,(\zeta^2+1)^{\frac{1}{2}s}\,\frac{d^s\mathrm{P}_n(\mu)}{d\mu^s}\,\frac{d^s p_n(\zeta)}{d\zeta^s}\,\begin{matrix}\cos\\ \sin\end{matrix}\Big\}\, s\omega, \quad . \quad . \quad (9.)$$

where P_n is the zonal harmonic, and p_n a similar function in which all the terms are $+$, instead of alternately $+$ and $-$.*

The terms for which $s = 0$ are symmetrical about the axis, and produce no currents, but only a certain superficial electrification. The density of this is calculated for the particular case $n = 1$, *i.e.*, for the case of a disk rotating in a *uniform* field about an axis parallel to the lines of force.

The only terms of the expansion (9) which produce sensible currents in a rotating disk are those tessaral solid harmonics for which $n-s$ is odd. The induced current-function is found to be (taking, say, $\cos s\omega$ in (9))

$$\phi = -\frac{2\,.\,4\,.\,.\,.\,(n+s-1)}{1\,.\,3\,.\,.\,.\,(n-s)}\,\frac{\overline{\mathrm{A}}}{\pi^2}\sin\eta\,(1-\mu^2)^{\frac{1}{2}s}\,\frac{d^s\mathrm{P}_n(\mu)}{d\mu^s}\sin\,(s\omega-\eta), \tag{10.}$$

where
$$\eta = \text{arc}\tan sp\tau,$$

τ having the value (7).

The most important type of induced currents is when $n = 2$, $s = 1$; in which case

$$\overline{\Omega} \propto xz,$$

* See Ferrers, 'Spherical Harmonics,' chap. vi.

so that the lines of force at the disk are normal to it, but the direction of the force is reversed as we cross the axis of y. The current-function relatively to axes displaced through the proper angle η in the direction of rotation, varies as

$$y\sqrt{\{1-r^2/a^2\}}.$$

A drawing of the current lines for this case is given. As already mentioned, they are simply the orthogonal projections of the contour lines of the tessaral harmonic of the second order.

In the next type we have $n = 3$, $s = 2$, so that

$$\bar{\Omega} \propto z(x^2-y^2),$$

and the current-function, relatively to displaced axes as before, varies as

$$xy\sqrt{\{1-r^2/a^2\}}.$$

IV. "On the Magnetisation of Iron in Strong Fields." By Professor J. A. Ewing, B.Sc., F.R.S.E., University College, Dundee, and Mr. William Low. Communicated by Sir W. Thomson, Knt., LL.D., F.R.S. Received March 2, 1887.

(Plate 2.)

The behaviour of iron and steel when subjected to very strong magnetising forces is a matter of considerable practical and very great theoretical interest, especially from its bearing on the molecular theory of magnetisation, which assigns an upper limit to the intensity of magnetism that a piece of iron can acquire, and even suggests that the metal may become diamagnetic under the influence of a sufficiently great force. All experiments hitherto made, by magnetising iron in the field of an electric solenoid, have shown that the intensity of magnetism $\mathfrak{I}$, as well as the induction $\mathfrak{B}$, is increasing with the highest values actually given to the magnetising force $\mathfrak{H}$. It is scarcely practicable, however, to produce by the direct action of a magnetising solenoid, a field whose force exceeds a few hundreds of C.G.S. units.

To refer to a few recent experiments of this class:—In experiments by one of us* on the magnetisation of long wires, the highest value of $\mathfrak{H}$ applied to iron was about 90, and this gave an induction $\mathfrak{B}$ of 16,500 in a soft iron wire. In Dr. Hopkinson's experiments† a force

* Ewing, "Exp. Res. in Magnetism," 'Phil. Trans.,' 1885, Part II.

† J. Hopkinson, "Magnetisation of Iron," 'Phil. Trans.,' 1885, Part II.